Funzioni trigonometriche Esercizi #4

(Integrali indefiniti elementari) Calcolo integrale

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

Esercizi

Calcolare

1. $\int \cos^2(2x)\,dx = \left[\frac{1}{2}x + \frac{1}{8}\sin(4x) + C\right]$
2. $\int \frac{1}{\sin x \cos x}\,dx = \left[-\ln|\cos x| + \ln|\sin x| + C\right]$
3. $\int \tan^2 x\,dx = \left[\tan x - x + C\right]$
4. $\int \frac{1}{\sin^2 x \cos^2 x}\,dx = \left[\tan x - \cot x + C\right]$
5. $\int 4\cos^4 x\,dx = \left[\frac{3}{2}x + \sin(2x) + \frac{1}{8}\sin(4x) + C\right]$
6. $\int \cos^3 x\,dx = \left[\sin x - \frac{1}{3}\sin^3 x + C\right]$
7. $\int \sin^5 x\,dx = \left[-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C\right]$
8. $\int \sqrt{1+\cos x}\,dx = \left[2\sqrt{2}\sin\frac{x}{2} + C\right]$
9. $\int (1-\cos(2x))^{\frac{3}{2}}\,dx = \left[2\sqrt{2}\left(-\cos x + \frac{\cos^3 x}{3}\right) + C\right]$

Se vi piace iscrivetevi al canale, mettete un mi piace o lasciate un commento

Soluzione

Esercizio 1

> Calcolare $I = \int \cos^2(2x)\, dx$

Soluzione

Da $1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$ si ha

$$
\begin{aligned}
I &= \int \frac{1+\cos(4x)}{2}\, dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(4x)\, dx \\
&= \frac{1}{2}x + \frac{1}{8}\int 4\cos(4x)\, dx \\
&= \frac{1}{2}x + \frac{1}{8}\sin(4x) + C
\end{aligned}
$$

Esercizio 2

Calcolare $I = \displaystyle\int \frac{1}{\sin x \cos x}\, dx$

Soluzione

$$
\begin{aligned}
I &= \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\, dx = \int \frac{\sin x}{\cos x}\, dx + \int \frac{\cos x}{\sin x}\, dx \\
&= -\int \frac{-\sin x}{\cos x}\, dx + \int \frac{\cos x}{\sin x}\, dx \\
&= -\ln|\cos x| + \ln|\sin x| + C
\end{aligned}
$$

Esercizio 3

Calcolare $I = \int \tan^2 x \, dx$

Soluzione

$$
\begin{aligned}
I &= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx \\
&= \int \frac{1}{\cos^2 x} \, dx - \int dx \\
&= \tan x - x + C
\end{aligned}
$$

Esercizio 4

Calcolare $I = \displaystyle\int \frac{1}{\sin^2 x \cos^2 x}\, dx$

Soluzione

$$\begin{aligned} I &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx \\ &= \int \frac{1}{\cos^2 x}\, dx + \int \frac{1}{\sin^2 x}\, dx \\ &= \tan x - \cot x + C \end{aligned}$$

Esercizio 5

> Calcolare $I = \int 4\cos^4 x\, dx$

Soluzione

Da $1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$ si ha

$$
\begin{aligned}
I &= \int 4\left(\frac{1+\cos(2x)}{2}\right)^2 dx = \int dx + 2\int \cos(2x)\, dx + \int \cos^2(2x)\, dx \\
&= x + \sin(2x) + \int \left(\frac{1+\cos(4x)}{2}\right) dx \\
&= x + \sin(2x) + \frac{1}{2}\int dx + \frac{1}{2}\int \cos(4x)\, dx = x + \sin(2x) + \frac{1}{2}x + \frac{1}{8}\int 4\cos(4x)\, dx \\
&= \frac{3}{2}x + \sin(2x) + \frac{1}{8}\sin(4x) + C
\end{aligned}
$$

Esercizio 6

Calcolare $I = \int \cos^3 x \, dx$

Soluzione

$$
\begin{aligned}
I &= \int (1 - \sin^2 x) \cos x \, dx = \int \cos x \, dx - \int \sin^2 x \cos x \, dx \\
&= \sin x - \int \sin^2 x \cos x \, dx \\
&= \begin{pmatrix} u = \sin x \\ du = \cos x \, dx \end{pmatrix} = \sin x - \int u^2 \, du = \sin x - \frac{u^3}{3} \\
&= (u = \sin x) = \sin x - \frac{1}{3} \sin^3 x + C
\end{aligned}
$$

Esercizio 7

Calcolare $I = \int \sin^5 x \, dx$

Soluzione

$$
\begin{aligned}
I &= \int \sin^4 x \sin x \, dx = \int \left(1 - \cos^2 x\right)^2 \sin x \, dx \\
&= \int \left(1 - 2\cos^2 x + \cos^4 x\right) \sin x \, dx \\
&= \int \sin x \, dx - 2 \int \cos^2 x \sin x \, dx + \int \cos^4 x \sin x \, dx \\
&= -\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C
\end{aligned}
$$

Esercizio 8

Calcolare $I = \int \sqrt{1 + \cos x}\, dx$

Soluzione

Da $1 + \cos\alpha = 2\cos^2 \frac{\alpha}{2}$ si ha

$$\begin{aligned} I &= \sqrt{2} \int \cos \frac{x}{2}\, dx = 2\sqrt{2} \int \frac{1}{2} \cos \frac{x}{2}\, dx \\ &= 2\sqrt{2} \sin \frac{x}{2} + C \end{aligned}$$

Esercizio 9

> Calcolare $I = \int (1 - \cos 2x)^{\frac{3}{2}}\, dx$

Soluzione

Da $1 - \cos\alpha = 2\sin^2 \frac{\alpha}{2}$ si ha

$$
\begin{aligned}
I &= \int \left(2\sin^2 x\right)^{\frac{3}{2}} dx = 2\sqrt{2}\int \sin^3 x\, dx \\
&= 2\sqrt{2}\int \left(1 - \cos^2 x\right)\sin x\, dx \\
&= 2\sqrt{2}\left(\int \sin x\, dx - \int \cos^2 x \sin x\, dx\right) \\
&= 2\sqrt{2}\left(-\cos x + \frac{\cos^3 x}{3}\right) + C
\end{aligned}
$$