Funzioni trigonometriche Esercizi #3

(Integrali indefiniti elementari) Calcolo integrale

Manolo Venturin

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

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Esercizi

Calcolare

1. $\int 2\cos x\, e^{\sin x}\, dx \qquad = \quad \left[2e^{\sin x} + C\right]$
2. $\int \cot x\, dx \qquad = \quad \left[\ln\left|\sin x\right| + C\right]$
3. $\int \frac{2}{x^2\left(1+\cos\frac{2}{x}\right)}\, dx \qquad = \quad \left[-\tan\frac{1}{x} + C\right]$
4. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx \qquad = \quad \left[-2\cos\left(\sqrt{x}\right) + C\right]$
5. $\int (1+2\sin x)^5 \cos x\, dx \qquad = \quad \left[\frac{1}{12}(1+2\sin x)^6 + C\right]$
6. $\int \tan^3 x \cdot \frac{1}{\cos^2 x}\, dx \qquad = \quad \left[\frac{(\tan x)^4}{4} + C\right]$
7. $\int \frac{1}{1+\sin x} \cos x\, dx \qquad = \quad \left[\ln\left|1+\sin x\right| + C\right]$
8. $\int \frac{\cos(\ln x)}{x}\, dx \qquad = \quad \left[\sin(\ln x) + C\right]$
9. $\int \frac{\sin(\ln x)}{x}\, dx \qquad = \quad \left[-\cos(\ln x) + C\right]$

Se vi piace iscrivetevi al canale, mettete un mi piace o lasciate un commento

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Soluzione

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Esercizio 1

> Calcolare $I = \displaystyle\int 2\cos x\, e^{\sin x}\, dx$

Soluzione

$$\begin{aligned} I &= \begin{pmatrix} u = \sin x \\ du = \cos x\, dx \end{pmatrix} = 2\int e^u\, du \\ &= e^u \\ &= (\, u = \sin x \,) = 2e^{\sin x} + C \end{aligned}$$

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Esercizio 2

> Calcolare $I = \int \cot x \, dx$

Soluzione

$$
\begin{aligned}
I &= \int \frac{\cos x}{\sin x}\, dx = \begin{pmatrix} u = \sin x \\ du = \cos x \, dx \end{pmatrix} \\
&= \int \frac{1}{u}\, du = \ln |u| \\
&= (\, u = \sin x \,) = \ln |\sin x| + C
\end{aligned}
$$

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Esercizio 3

Calcolare $\quad I = \displaystyle\int \frac{2}{x^2\left(1+\cos\frac{2}{x}\right)}\,dx$

Soluzione

Da $1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$ si ha

$$1+\cos\frac{2}{x} = 2\cos^2\frac{\cancel{2}}{\cancel{2}\,x} = 2\cos^2\frac{1}{x}$$

e quindi

$$\begin{aligned} I &= \int \frac{\cancel{2}}{\cancel{2}\,x^2\cos^2\frac{1}{x}}\,dx = \begin{pmatrix} u = \frac{1}{x} \\ du = -\frac{1}{x^2}\,dx \end{pmatrix} = -\int \frac{1}{\cos^2 u}\,du \\ &= -\tan u = \left(u = \frac{1}{x}\right) = -\tan\frac{1}{x} + C \end{aligned}$$

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Esercizio 4

> Calcolare $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx \end{pmatrix} = 2\int \sin u\,du \\
&= -2\cos u \\
&= \left(u = \sqrt{x} \right) = -2\cos\left(\sqrt{x}\right) + C
\end{aligned}
$$

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Esercizio 5

> Calcolare $\quad I = \int (1 + 2\sin x)^5 \cos x \, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = 1 + 2\sin x \\ du = 2\cos x \, dx \end{pmatrix} = \frac{1}{2}\int u^5 \, du \\
&= \frac{1}{2}\frac{u^6}{6} \\
&= (\, u = 1 + 2\sin x \,) = \frac{1}{12}(1 + 2\sin x)^6 + C
\end{aligned}
$$

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Esercizio 6

> Calcolare $I = \displaystyle\int \tan^3 x \cdot \frac{1}{\cos^2 x}\, dx$

Soluzione

$$\begin{aligned}
I &= \begin{pmatrix} u = \tan x \\ du = \dfrac{1}{\cos^2 x}\, dx \end{pmatrix} = \int u^3\, du \\
&= \frac{u^4}{4} \\
&= (\, u = \tan x\,) = \frac{(\tan x)^4}{4} + C
\end{aligned}$$

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Esercizio 7

> Calcolare $I = \int \frac{1}{1+\sin x} \cos x \, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = 1+\sin x \\ du = \cos x \, dx \end{pmatrix} = \int \frac{1}{u} \, du \\
&= \ln |u| \\
&= \left(u = 1+\sin x \right) = \ln |1+\sin x| + C
\end{aligned}
$$

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Esercizio 8

> Calcolare $I = \int \frac{\cos(\ln x)}{x}\, dx$

Soluzione

$$\begin{aligned} I &= \begin{pmatrix} u = \ln x \\ du = \frac{1}{x}\, dx \end{pmatrix} = \int \cos u \, du \\ &= \sin u \\ &= (\, u = \ln x \,) = \sin(\ln x) + C \end{aligned}$$

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Esercizio 9

> Calcolare $I = \int \frac{\sin(\ln x)}{x}\,dx$

Soluzione

$$\begin{aligned} I &= \begin{pmatrix} u = \ln x \\ du = \frac{1}{x}\,dx \end{pmatrix} = \int \sin u\,du \\ &= -\cos u \\ &= (\,u = \ln x\,) = -\cos(\ln x) + C \end{aligned}$$

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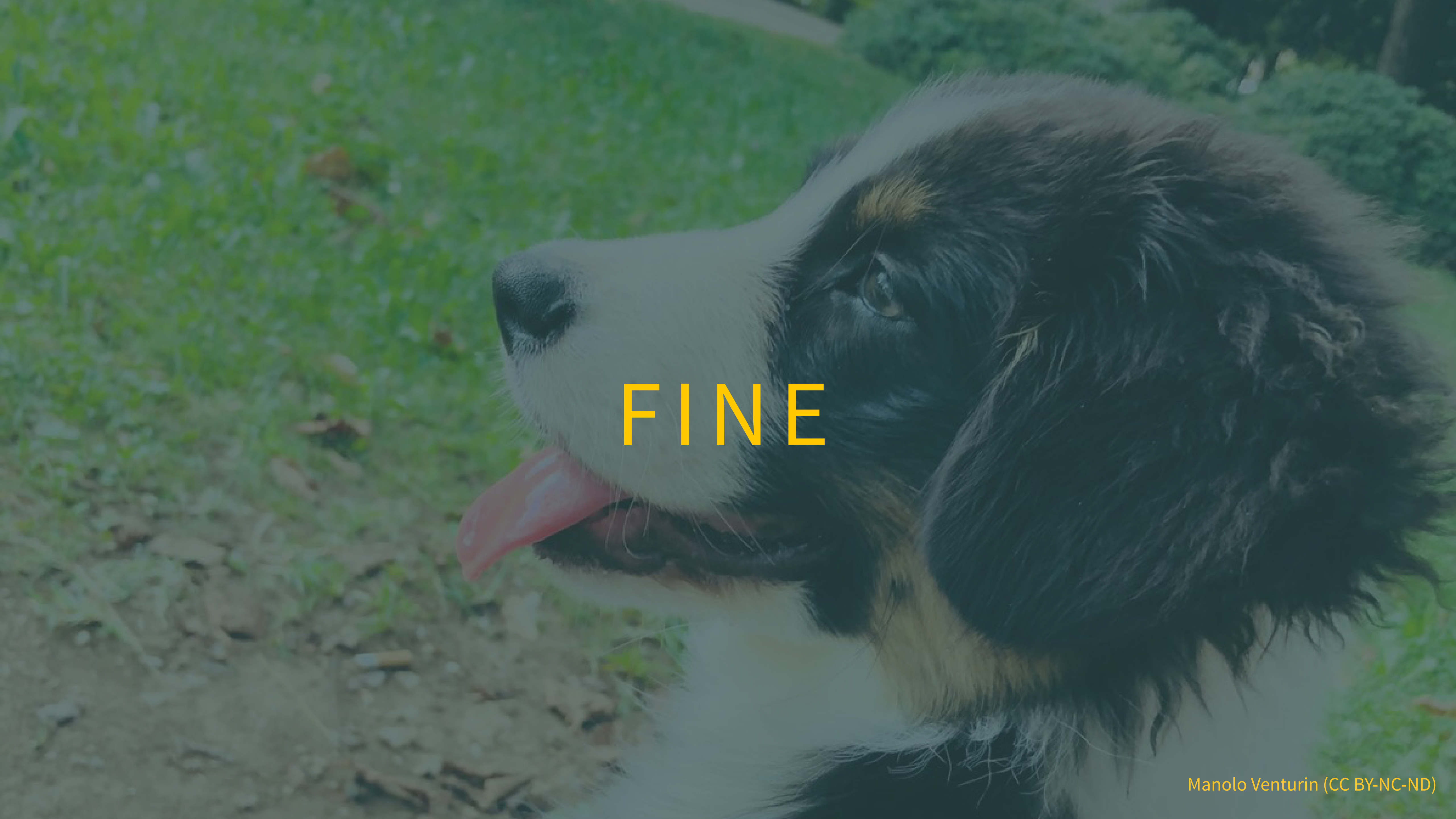
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