Funzioni trigonometriche Esercizi #2

(Integrali indefiniti elementari) Calcolo integrale

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

Esercizi

Calcolare

1. $\int 12x^2 \sin\left(1+2x^3\right) dx = \left[-2\cos\left(1+2x^3\right)+C\right]$
2. $\int \cos^3 x \, dx = \left[\sin x - \frac{\sin^3 x}{3} + C\right]$
3. $\int (3x^2+2x)\cos(x^3+x^2+1)\, dx = \left[\sin\left(x^3+x^2+1\right)+C\right]$
4. $\int \frac{\sin x}{\cos^3 x}\, dx = \left[\frac{1}{2}\frac{1}{\cos^2 x}+C\right]$
5. $\int 4x \sin\left(2x^2\right) dx = \left[-\cos\left(2x^2\right)+C\right]$
6. $\int (1+\sin x)^2 \cos x \, dx = \left[\frac{1}{3}(1+\sin x)^3+C\right]$
7. $\int \frac{1}{\sqrt{\cos x}} \sin x \, dx = \left[-2\sqrt{\cos x}+C\right]$
8. $\int \frac{1+\cos x}{x+\sin x}\, dx = \left[\ln|x+\sin x|+C\right]$
9. $\int \frac{\cos x}{\sin^3 x}\, dx = \left[-\frac{1}{2}\frac{1}{\sin^2 x}+C\right]$

Se vi piace iscrivetevi al canale, mettete un mi piace o lasciate un commento

Soluzione

Esercizio 1

Calcolare $I = \int 12x^2 \sin\left(1 + 2x^3\right) dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = 1 + 2x^3 \\ du = 6x^2\, dx \end{pmatrix} = 2 \int \sin u \, du \\
&= -2 \cos u \\
&= \left(u = 1 + 2x^3 \right) = -2 \cos\left(1 + 2x^3\right) + C
\end{aligned}
$$

Esercizio 2

> Calcolare $I = \int \cos^3 x \, dx$

Soluzione

$$I = \int \cos^2 x \cos x \, dx = \int (1 - \sin^2 x) \cos x \, dx = \underbrace{\int \cos x \, dx}_{A} - \underbrace{\int \sin^2 x \cos x \, dx}_{B} = A - B$$

- $A = \int \cos x \, dx = \sin x$
- $B = \int \sin^2 x \cos x \, dx = \begin{pmatrix} u = \sin x \\ du = \cos x \, dx \end{pmatrix} = \int u^2 \, du = \frac{u^3}{3} = \frac{\sin^3 x}{3}$
- $A - B = \sin x - \frac{\sin^3 x}{3} + C$

Esercizio 3

Calcolare $I = \int (3x^2 + 2x)\cos(x^3 + x^2 + 1)\,dx$

Soluzione

$$
\begin{aligned}
I &= \int x(3x+2)\cos(x^3+x^2+1)\,dx \\
&= \left(\begin{array}{c} u = x^3 + x^2 + 1 \\ du = (3x^2 + 2x)\,dx \end{array}\right) = \int \cos u\,du \\
&= \sin u \\
&= (\,u = x^3 + x^2 + 1\,) = \sin(x^3 + x^2 + 1) + C
\end{aligned}
$$

Esercizio 4

Calcolare $I = \displaystyle\int \frac{\sin x}{\cos^3 x}\, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = \cos x \\ du = -\sin x\, dx \end{pmatrix} = -\int \frac{1}{u^3}\, du \\
&= -\int u^{-3}\, du = -\frac{u^{-2}}{-2} \\
&= (\, u = \cos x \,) = \frac{1}{2}\frac{1}{\cos^2 x} + C
\end{aligned}
$$

Esercizio 5

Calcolare $I = \int 4x \sin\left(2x^2\right) dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = 2x^2 \\ du = 4x\,dx \end{pmatrix} = \int \sin u \, du \\
&= -\cos u \\
&= (\, u = 2x^2 \,) = -\cos\left(2x^2\right) + C
\end{aligned}
$$

Esercizio 6

Calcolare $I = \int (1 + \sin x)^2 \cos x \, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = 1 + \sin x \\ du = \cos x \, dx \end{pmatrix} = \int u^2 \, du \\
&= \frac{1}{3} u^3 \\
&= (\, u = 1 + \sin x \,) = \frac{1}{3} (1 + \sin x)^3 + C
\end{aligned}
$$

Esercizio 7

Calcolare $I = \displaystyle\int \frac{1}{\sqrt{\cos x}} \sin x \, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = \cos x \\ du = -\sin x \, dx \end{pmatrix} = -\int \frac{1}{\sqrt{u}} \, du \\
&= -\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \\
&= (\, u = \cos x \,) = -2\sqrt{\cos x} + C
\end{aligned}
$$

Esercizio 8

Calcolare $I = \displaystyle\int \frac{1+\cos x}{x+\sin x}\,dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = x+\sin x \\ du = (1+\cos x)\,dx \end{pmatrix} = \int \frac{1}{u}\,du \\
&= \ln|u| \\
&= (\,u = x+\sin x\,) = \ln|x+\sin x| + C
\end{aligned}
$$

Esercizio 9

Calcolare $I = \int \frac{\cos x}{\sin^3 x}\, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = \sin x \\ du = \cos x\, dx \end{pmatrix} = \int \frac{1}{u^3}\, du \\
&= \int u^{-3}\, du = \frac{u^{-2}}{-2} \\
&= (u = \sin x) = -\frac{1}{2}\frac{1}{\sin^2 x} + C
\end{aligned}
$$