Esponenziale Esercizi #2

(Integrali indefiniti elementari) Calcolo integrale

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

Esercizi

1. Calcolare $\int \frac{e^{2x}-1}{e^{2x}+2}\,dx$ $= \left[-\frac{x}{2}+\frac{3}{4}\ln\left(e^{2x}+2\right)+C\right]$
2. $\int \frac{2e^x}{e^x+e^{-x}}\,dx$ $= \left[\ln\left|e^{2x}+1\right|+C\right]$
3. $\int \frac{(1+2\ln x)}{x}\,dx$ $= \left[\ln x+\ln^2 x+C\right]$
4. $\int \frac{1}{x\sqrt{\ln x}}\,dx$ $= \left[2\sqrt{\ln x}+C\right]$
5. $\int \frac{1}{x\ln x}\,dx$ $= \left[\ln\left|\ln x\right|+C\right]$
6. $\int (1+e^x)^2 e^x\,dx$ $= \left[\frac{1}{3}(1+e^x)^3+C\right]$
7. $\int \frac{e^{2x}}{1+e^{2x}}\,dx$ $= \left[\frac{1}{2}\ln\left(1+e^{2x}\right)+C\right]$
8. $\int \frac{e^{-x}}{e^x+1}\,dx$ $= \left[\ln\left|e^x+1\right|-x-e^{-x}\right]$
9. $\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx$ $= \left[(\sqrt{x}+1)^2+2\ln\left|\sqrt{x}-1\right|+C\right]$
10. $\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx$ $= \left[\frac{2}{3}\sqrt{e^x-1}\,(e^x+2)+C\right]$

Soluzione

Se vi piace iscrivetevi al canale, mettete un mi piace o lasciate un commento

Esercizio 1

> Calcolare $\quad I = \displaystyle\int \frac{e^{2x}-1}{e^{2x}+2}\,dx$

Soluzione

$$I = \int \frac{e^{2x}+2-3}{e^{2x}+2}\,dx = \underbrace{\int dx}_{A} - \underbrace{3\int \frac{1}{e^{2x}+2}\,dx}_{B} = A - 3B$$

- $A = \int dx = x$
- $B = \int \frac{1}{e^{2x}+2}\,dx = \int \frac{e^{-2x}}{1+2e^{-2x}}\,dx = \left(\begin{array}{c} u = 1+2e^{-2x} \\ du = -4e^{-2x}\,dx \end{array}\right) = -\frac{1}{4}\int \frac{1}{u}\,du$

  $= -\frac{1}{4}\ln|u| = -\frac{1}{4}\ln\left(1+2e^{-2x}\right)$

Esercizio 1

- $I = A - 3B$
- $A = x$
- $B = -\frac{1}{4}\ln\left(1 + 2e^{-2x}\right)$

- $A - 3B = x + \frac{3}{4}\ln\left(1 + 2e^{-2x}\right) = x + \frac{3}{4}\ln\left(\frac{e^{2x}+2}{e^{2x}}\right)$

  $= x + \frac{3}{4}\ln\left(e^{2x} + 2\right) - \frac{3}{4}2x = -\frac{x}{2} + \frac{3}{4}\ln\left(e^{2x} + 2\right) + C$

Esercizio 2

> Calcolare $I = \int \frac{2e^x}{e^x + e^{-x}}\,dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} u = e^x \\ du = e^x\,dx \end{pmatrix} = \int \frac{2}{u + \frac{1}{u}}\,du \\
&= \int \frac{2u}{u^2+1} = \ln\left|u^2+1\right| \\
&= \ln\left|e^{2x}+1\right| + C
\end{aligned}
$$

Esercizio 3

> Calcolare $\quad I = \displaystyle\int \frac{(1+2\ln x)}{x}\,dx$

Soluzione

$$I \;=\; \int \frac{1}{x}\,dx + 2\int \frac{\ln x}{x}\,dx \;=\; \ln x + \ln^2 x + C$$

oppure

$$I \;=\; \begin{pmatrix} u = 1 + 2\ln x \\ du = \frac{2}{x}\,dx \end{pmatrix} \;=\; \frac{1}{2}\int u\,du \;=\; \frac{1}{2}\,\frac{u^2}{2} \;=\; \frac{1}{4}(1+2\ln x)^2 + C$$

Esercizio 4

> Calcolare $I = \int \frac{1}{x\sqrt{\ln x}}\, dx$

Soluzione

$$I = \begin{pmatrix} u = \ln x \\ du = \frac{1}{x}\, dx \end{pmatrix} = \int \frac{1}{\sqrt{u}}\, du = \int u^{-\frac{1}{2}}\, du = 2u^{\frac{1}{2}} = 2\sqrt{\ln x} + C$$

Esercizio 5

> Calcolare $I = \int \frac{1}{x \ln x} \, dx$

Soluzione

$$I = \left(\begin{array}{c} u = \ln x \\ du = \frac{1}{x} \, dx \end{array} \right) = \int \frac{1}{u} \, du = \ln |u| = \ln | \ln x | + C$$

Esercizio 6

> Calcolare $I = \int (1+e^x)^2 e^x \, dx$

Soluzione

$$I = \begin{pmatrix} u = 1+e^x \\ du = e^x \, dx \end{pmatrix} = \int u^2 \, du = \frac{u^3}{3} = \frac{1}{3}(1+e^x)^3 + C$$

Esercizio 7

> Calcolare $I = \displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$

Soluzione

$$I = \begin{pmatrix} u = e^{2x} \\ du = 2e^{2x}\,dx \end{pmatrix} = \frac{1}{2}\int \frac{1}{1+u}\,du = \frac{1}{2}\ln|1+u| = \frac{1}{2}\ln\left(1+e^{2x}\right) + C$$

Esercizio 8

Calcolare $\quad I = \displaystyle\int \frac{e^{-x}}{e^x+1}\,dx$

Soluzione

$$I = \left(du = \underbrace{e^x}_{u}\, dx \overset{u=e^x}{\Longrightarrow} \tfrac{1}{u} du = dx \right) = \int \frac{\frac{1}{u}}{u+1}\frac{1}{u}\,du = \int \frac{1}{u^2(u+1)}\,du$$

Da

$$\frac{1}{u^2(u+1)} = \frac{(u+1)-u}{u^2(u+1)} = \frac{1}{u^2} - \frac{1}{u(u+1)} = \frac{1}{u^2} - \frac{(u+1)-u}{u(u+1)} = \frac{1}{u^2} - \frac{1}{u} + \frac{1}{u+1}$$

si ha

$$\int \frac{1}{u^2(u+1)}\,du = \int \frac{1}{u^2}\,du - \int \frac{1}{u}\,du + \int \frac{1}{u+1}\,du = \ln|u+1| - \ln|u| - \frac{1}{u} = \ln|e^x+1| - x - e^{-x}$$

Esercizio 8

Eventualmente

$$I = \ln|e^x + 1| - x - e^{-x} + C$$

può essere riscritto come

$$\ln|e^x + 1| - \ln|e^x| - e^{-x} = \ln\left|\frac{e^x + 1}{e^x}\right| - e^{-x} = \ln\left|1 + e^{-x}\right| - e^{-x} + C$$

Esercizio 9

> Calcolare $I = \int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx$

Soluzione

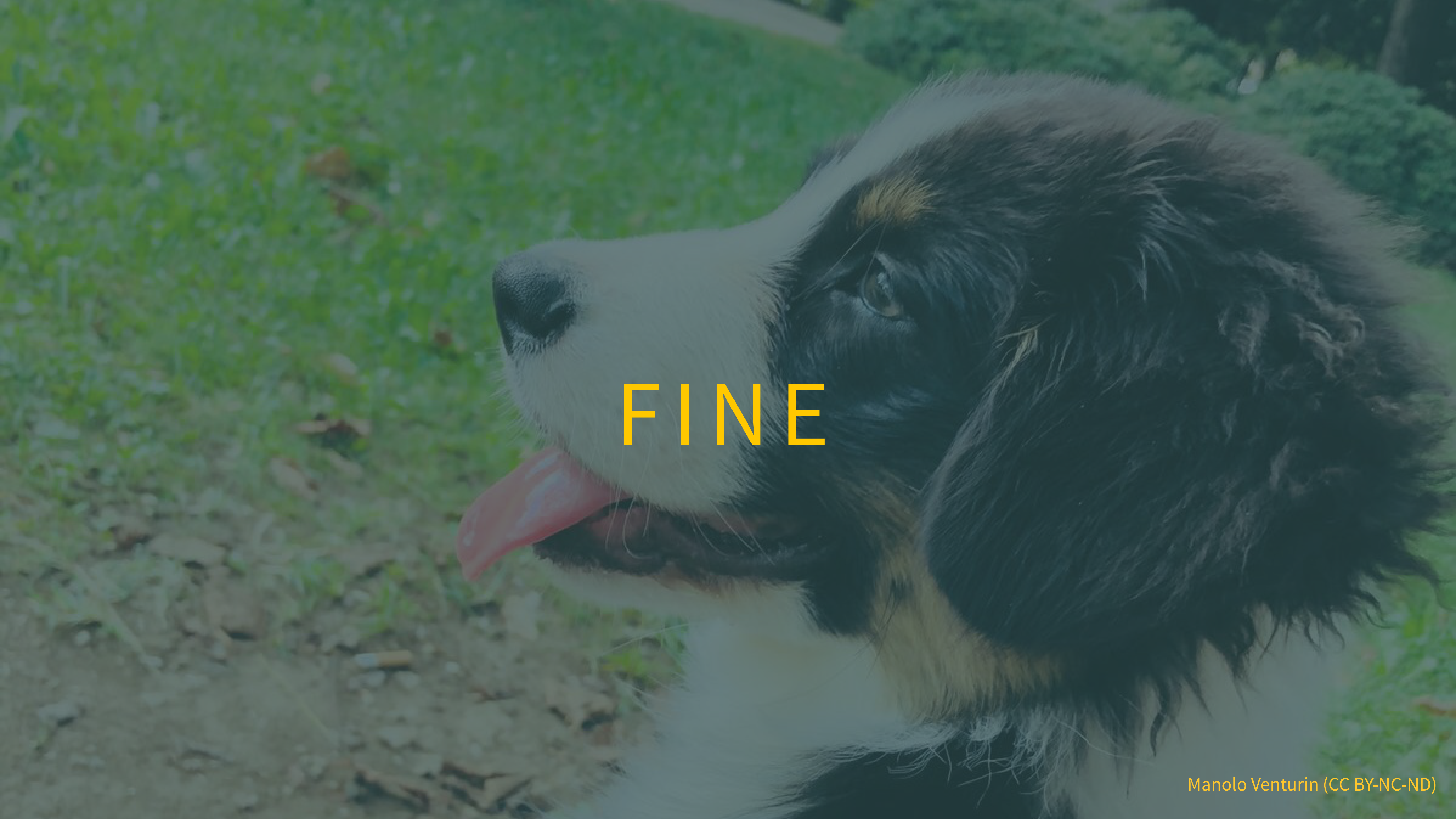
$$
\begin{aligned}
I &= \left(\begin{array}{c} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx \implies 2u\,du = dx \end{array} \right) = \int \frac{u}{u-1} 2u\,du = 2\int \frac{u^2}{u-1}\,du \\
&= 2\int \frac{(u^2-1)+1}{u-1}\,du = 2\int \frac{(u-1)(u+1)+1}{u-1}\,du \\
&= 2\int (u+1)\,du + 2\int \frac{1}{u-1}\,du = \cancel{2}\,\frac{(u+1)^2}{\cancel{2}} + 2\ln|u-1| \\
&= (u = \sqrt{x}) = (\sqrt{x}+1)^2 + 2\ln|\sqrt{x}-1| + C
\end{aligned}
$$

Esercizio 10

> Calcolare $\quad I = \displaystyle\int \frac{e^{2x}}{\sqrt{e^x - 1}}\, dx$

Soluzione

$$
\begin{aligned}
I &= \begin{pmatrix} & u = e^x & \\ du = e^x\, dx & \implies & \frac{1}{u} du = dx \end{pmatrix} = \int \frac{u^2}{\sqrt{u-1}} \frac{1}{u}\, du = \int \frac{u}{\sqrt{u-1}}\, du \\
&= \int \frac{(u-1)+1}{\sqrt{u-1}}\, du = \int \sqrt{u-1}\, du + \int \frac{1}{\sqrt{u-1}}\, du \\
&= \frac{(u-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(u-1)^{-\frac{1}{2}+1}}{\frac{-1}{2}+1} = \frac{2}{3}(u-1)^{\frac{3}{2}} + 2(u-1)^{\frac{1}{2}} = \frac{2}{3}(u-1)^{\frac{1}{2}}\left((u-1)+3\right) \\
&= \left(\, u = e^x \,\right) = \frac{2}{3}\sqrt{e^x - 1}\,(e^x + 2) + C
\end{aligned}
$$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