

Le potenze (versione generalizzata) Esercizi #2

(Integrali indefiniti elementari) Calcolo integrale

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# Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!



# Esercizi

1. Calcolare  $\int \frac{-x}{\left(\frac{1}{2}x^2+10\right)^{10}} dx$

$$= \left[ \frac{1}{9} \frac{1}{\left(\frac{1}{2}x^2+10\right)^9} + C \right]$$

2.  $\int \sqrt[3]{(x-1)^2} dx$

$$= \left[ \frac{3}{5} (x-1)^{\frac{5}{3}} + C \right]$$

3.  $\int \frac{x}{1+x^2} dx$

$$= \left[ \frac{1}{2} \ln(1+x^2) + C \right]$$

4.  $\int \frac{1}{\sqrt[3]{2x+1}} dx$

$$= \left[ \frac{3}{4} (2x+1)^{\frac{2}{3}} + C \right]$$

5.  $\int \frac{2x-1}{x^2-x+3} dx$

$$= \left[ \ln |x^2 - x + 3| + C \right]$$

6.  $\int \frac{1}{(3x-1)^3} dx$

$$= \left[ -\frac{1}{6} \frac{1}{(3x-1)^2} + C \right]$$

7.  $\int \frac{1}{x-\sqrt{x}} dx$

$$= \left[ 2 \ln |\sqrt{x} - 1| + C \right]$$

8.  $\int \frac{x}{1+x} dx$

$$= \left[ x - \ln |1+x| + C \right]$$

9.  $\int \frac{1}{x(1+x)} dx$

$$= \left[ \ln |x| - \ln |1+x| + C \right]$$

10.  $\int \frac{1-x^4+2x}{1+x^2} dx$

$$= \left[ x - \frac{x^3}{3} + \ln |1+x^2| + C \right]$$

# Soluzione

**Se vi piace iscrivetevi al canale, mettete un mi piace o lasciate un commento**

# Esercizio 1

Calcolare  $I = \int \frac{-x}{\left(\frac{1}{2}x^2 + 10\right)^{10}} dx$

Soluzione

$$\begin{aligned} I &= \left( \begin{array}{l} u = \frac{1}{2}x^2 + 10 \\ du = x dx \end{array} \right) = - \int u^{-10} du = -\frac{u^{-9}}{-9} \\ &= \frac{1}{9} \left( \frac{1}{2}x^2 + 10 \right)^{-9} = \frac{1}{9} \frac{1}{\left(\frac{1}{2}x^2 + 10\right)^9} + C \end{aligned}$$

# Esercizio 2

Calcolare  $I = \int \sqrt[3]{(x-1)^2} dx$

Soluzione

$$I = \left( \begin{array}{l} u = (x-1) \\ du = dx \end{array} \right) = \int u^{\frac{2}{3}} du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} = \frac{3}{5}(x-1)^{\frac{5}{3}} + C$$

# Esercizio 3

Calcolare  $I = \int \frac{x}{1+x^2} dx$

Soluzione

$$\begin{aligned} I &= \left( \begin{array}{l} u = 1 + x^2 \\ du = 2x \, dx \end{array} \right) = \frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \int \frac{1}{u} du \\ &= \frac{1}{2} \ln |u| = \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

# Esercizio 4

Calcolare  $I = \int \frac{1}{\sqrt[3]{2x+1}} dx$

**Soluzione**

$$\begin{aligned} I &= \left( \begin{array}{l} u = 2x + 1 \\ du = 2x \, dx \end{array} \right) = \frac{1}{2} \int \frac{1}{\sqrt[3]{2x+1}} 2 \, dx = \frac{1}{2} \int \frac{1}{\sqrt[3]{u}} \, du \\ &= \frac{1}{2} \frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} = \frac{1}{2} \frac{u^{\frac{2}{3}}}{\frac{2}{3}} = \frac{3}{4} (2x+1)^{\frac{2}{3}} + C \end{aligned}$$



# Esercizio 5

Calcolare  $I = \int \frac{2x - 1}{x^2 - x + 3} dx$

Soluzione

$$\begin{aligned} I &= \left( \begin{array}{l} u = x^2 - x + 3 \\ du = (2x - 1) dx \end{array} \right) = \int \frac{1}{u} du \\ &= \ln |u| = \ln |x^2 - x + 3| + C \end{aligned}$$

# Esercizio 6

Calcolare  $I = \int \frac{1}{(3x - 1)^3} dx$

Soluzione

$$\begin{aligned} I &= \left( \begin{array}{l} u = 3x - 1 \\ du = 3 dx \end{array} \right) = \frac{1}{3} \int u^{-3} du \\ &= \frac{1}{3} \frac{u^{-2}}{-2} = -\frac{1}{6} \frac{1}{(3x - 1)^2} + C \end{aligned}$$

# Esercizio 7

Calcolare  $I = \int \frac{1}{x - \sqrt{x}} dx$

Soluzione

$$\begin{aligned} I &= \left( \begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}} dx = \frac{1}{2u} dx \end{array} \right) = \int \frac{2u}{u^2 - u} du \\ &= 2 \int \frac{1}{u - 1} du = 2 \ln |u - 1| = 2 \ln |\sqrt{x} - 1| + C \end{aligned}$$

# Esercizio 8

Calcolare  $I = \int \frac{x}{1+x} dx$

Soluzione

$$I = \int \frac{(x+1) - 1}{1+x} dx = \int dx - \int \frac{1}{1+x} dx = x - \ln |1+x| + C$$

# Esercizio 9

Calcolare  $I = \int \frac{1}{x(1+x)} dx$

**Soluzione**

$$I = \int \frac{(1+x) - x}{x(1+x)} dx = \int \frac{1}{x} dx - \int \frac{1}{1+x} dx = \ln|x| - \ln|1+x| + C$$

# Esercizio 10

Calcolare  $I = \int \frac{1 - x^4 + 2x}{1 + x^2} dx$

Soluzione

$$\begin{aligned} I &= \int \frac{(1 - x^2)(1 + x^2) + 2x}{1 + x^2} dx \\ &= \int (1 - x^2) dx + \int \frac{2x}{1 + x^2} dx \\ &= x - \frac{x^3}{3} + \ln(1 + x^2) + C \end{aligned}$$

A close-up profile of a dog's head, likely a Bernese Mountain Dog, with its tongue out. The image is overlaid with a semi-transparent teal filter. The word "FINE" is written in a bold, yellow, sans-serif font across the middle of the dog's face.

FINE