Le potenze (versione base) Esercizi #3

(Integrali indefiniti elementari) Calcolo integrale

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

Indice degli esercizi

1. Calcolare: $\int \frac{1}{2x}\,dx = \left[\frac{1}{2}\ln|x| + C\right]$
2. $\int \frac{2x^3-3x^2+3}{x}\,dx = \left[\frac{2}{3}x^3 - \frac{3}{2}x^2 + 3\ln|x| + C\right]$
3. $\int \left(\frac{x}{2}\sqrt{x} + \sqrt[3]{x}\right)\,dx = \left[\frac{1}{5}x^2\sqrt{x} + \frac{3}{4}x\sqrt[3]{x} + C\right]$
4. $\int \frac{1}{3x\sqrt[3]{x}}\,dx = \left[-\frac{1}{\sqrt[3]{x}} + C\right]$
5. $\int 11x^2\sqrt[3]{x^2}\,dx = \left[3x^3\sqrt[3]{x^2} + C\right]$
6. $-\int \frac{8}{x^3\sqrt[3]{x^2}}\,dx = \left[\frac{3}{x^2\sqrt[3]{x^2}} + C\right]$
7. $\int \frac{1}{6\sqrt[6]{x^5}}\,dx = \left[\sqrt[6]{x} + C\right]$
8. $\int \frac{2-3x}{\sqrt{x}}\,dx = \left[4\sqrt{x} - 2x\sqrt{x} + C\right]$
9. $\int \frac{\frac{\sqrt{x}}{x}-1}{\sqrt{x}}\,dx = \left[\ln|x| - 2\sqrt{x} + C\right]$
10. $\int \frac{x^3+2\sqrt{x}}{x\sqrt{x}}\,dx = \left[\frac{2}{5}x^2\sqrt{x} + 2\ln|x| + C\right]$

Soluzione

Esercizio 1

> Calcolare $I = \int \frac{1}{2x}\,dx$

Soluzione

$$I = \frac{1}{2}\int \frac{1}{x}\,dx = \frac{1}{2}\ln|x| + C$$

Esercizio 2

> Calcolare $I = \int \frac{2x^3 - 3x^2 + 3}{x}\, dx$

Soluzione

$$\begin{aligned}
\int \frac{2x^3 - 3x^2 + 3}{x}\, dx &= 2\int x^2\, dx - 3\int x\, dx + 3\int \frac{1}{x}\, dx \\
&= 2\frac{x^3}{3} - 3\frac{x^2}{2} + 3\ln|x| \\
&= \frac{2}{3}x^3 - \frac{3}{2}x^2 + 3\ln|x| + C
\end{aligned}$$

Esercizio 3

> Calcolare $I = \int \left(\frac{x}{2} \sqrt{x} + \sqrt[3]{x} \right) \, dx$

Soluzione

- $I \;=\; I_1 + I_2 \;=\; \int \frac{x}{2}\sqrt{x}\,dx + \int \sqrt[3]{x}\,dx$
- $I_1 \;=\; \frac{1}{2}\int x^{1+\frac{1}{2}}\,dx \;=\; \frac{1}{2}\int x^{\frac{3}{2}}\,dx \;=\; \frac{1}{2}\,\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \;=\; \frac{1}{2}\,\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \;=\; \frac{1}{5}x^{\frac{5}{2}} \;=\; \frac{1}{5}x^2\sqrt{x} + C$
- $I_2 \;=\; \int \sqrt[3]{x}\,dx \;=\; \int x^{\frac{1}{3}}\,dx \;=\; \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} \;=\; \frac{3}{4}x^{\frac{4}{3}} \;=\; \frac{3}{4}x\sqrt[3]{x} + C$
- $I \;=\; I_1 + I_2 \;=\; \frac{1}{5}x^2\sqrt{x} + \frac{3}{4}x\sqrt[3]{x} + C$

Esercizio 4

> Calcolare $I = \int \frac{1}{3x\sqrt[3]{x}}\, dx$

Soluzione

$$
\begin{aligned}
I &= \frac{1}{3} \int x^{-\frac{4}{3}}\, dx \\
&= \frac{1}{3} \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} \\
&= -\frac{1}{\sqrt[3]{x}} + C
\end{aligned}
$$

Esercizio 5

> Calcolare $I = \int 11x^2\sqrt[3]{x^2}\,dx$

Soluzione

$$\begin{aligned} I &= 11\int x^{\frac{8}{3}}\,dx \\ &= \cancel{11}\,\frac{x^{\frac{11}{3}}}{\frac{\cancel{11}}{3}} = 3x^{\frac{11}{3}} \\ &= 3x^3\sqrt[3]{x^2} + C \end{aligned}$$

Esercizio 6

> Calcolare $I = -\int \frac{8}{x^3 \sqrt[3]{x^2}}\, dx$

Soluzione

$$
\begin{aligned}
I &= -8 \int x^{-\frac{11}{3}}\, dx \\
&= -\cancel{8}\, \frac{x^{-\frac{8}{3}}}{-\frac{\cancel{8}}{3}} = \frac{3}{x^{\frac{8}{3}}} \\
&= \frac{3}{x^2 \sqrt[3]{x^2}} + C
\end{aligned}
$$

Esercizio 7

> Calcolare $I = \int \frac{1}{6\sqrt[6]{x^5}}\,dx$

Soluzione

$$I = \frac{1}{6}\int x^{-\frac{5}{6}}\,dx = \frac{1}{6}\,\frac{x^{\frac{1}{6}}}{\frac{1}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x} + C$$

Esercizio 8

> Calcolare $I = \int \frac{2-3x}{\sqrt{x}}\,dx$

Soluzione

$$\begin{aligned} I &= 2\int \frac{1}{\sqrt{x}\,dx} - 3\int \sqrt{x}\,dx \\ &= 2\frac{\sqrt{x}}{\frac{1}{2}} - \cancel{3}\,\frac{x^{\frac{3}{2}}}{\frac{\cancel{3}}{2}} \\ &= 4\sqrt{x} - 2x\sqrt{x} + C \end{aligned}$$

Esercizio 9

> Calcolare $I = \int \dfrac{\frac{\sqrt{x}}{x} - 1}{\sqrt{x}}\, dx$

Soluzione

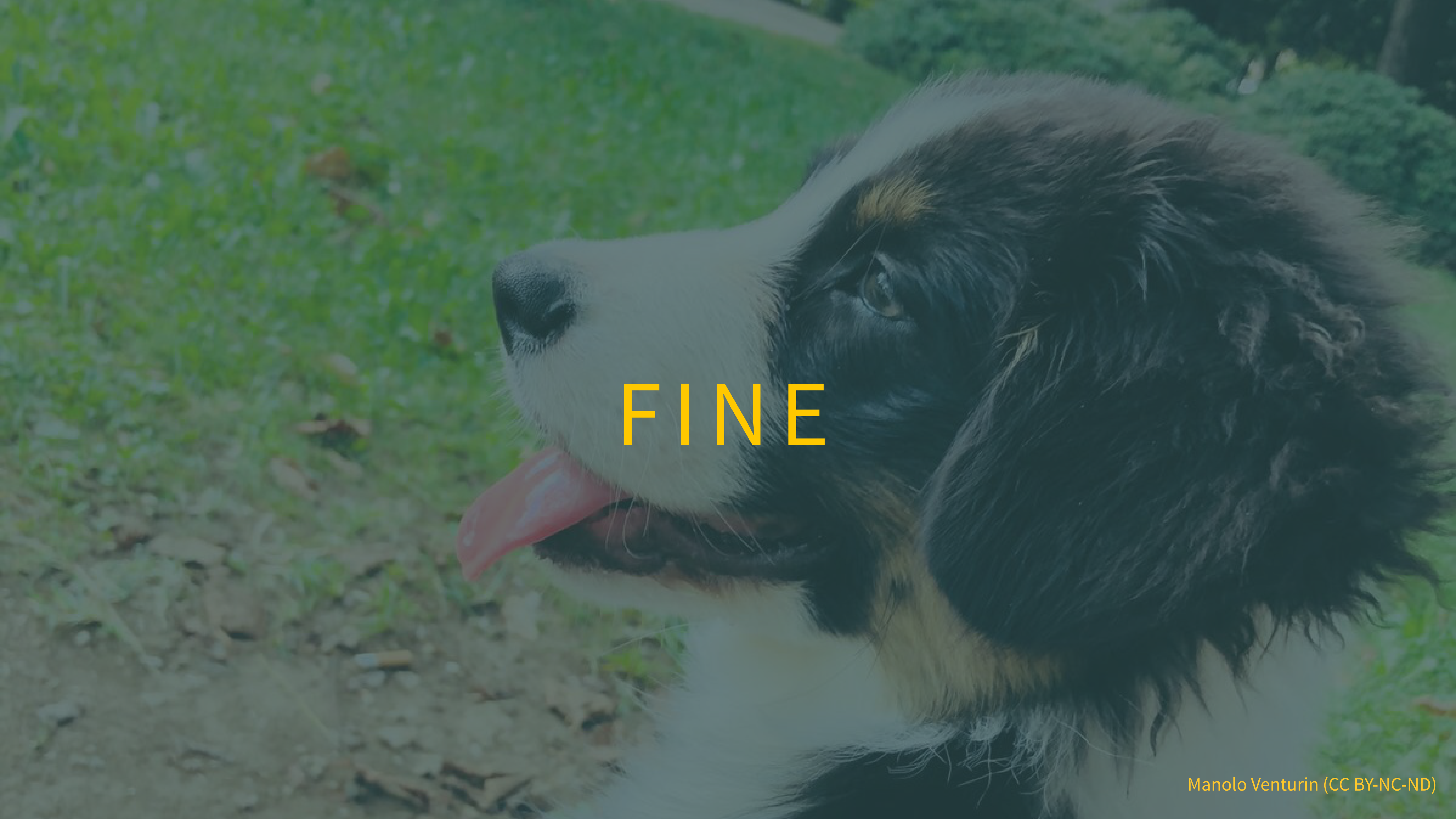
$$
\begin{aligned}
I &= \int \frac{1}{x}\, dx - \int \frac{1}{\sqrt{x}}\, dx \\
&= \ln|x| - \frac{\sqrt{x}}{\frac{1}{2}} \\
&= \ln|x| - 2\sqrt{x} + C
\end{aligned}
$$

Esercizio 10

> Calcolare $I = \int \frac{x^3 + 2\sqrt{x}}{x\sqrt{x}}\, dx$

Soluzione

$$
\begin{aligned}
I &= \int x\sqrt{x}\, dx + 2\int \frac{1}{x}\, dx \\
&= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2\ln|x| \\
&= \frac{2}{5}x^2\sqrt{x} + 2\ln|x| + C
\end{aligned}
$$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