Le potenze — Teoria

(Integrali indefiniti elementari) Calcolo integrale

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Donazione

Se apprezzi le mie slide, considera di fare una donazione per supportare il mio lavoro.

Grazie!

Indice degli esempi

Calcolare

1. $I = \displaystyle\int \left(\frac{1}{x^2} + \frac{1}{x} - 1 + 2x + 3x^2 + x^\pi \right)\, dx$
2. $I = \displaystyle\int x(1+x^2)^{999}\, dx$
3. $I = \displaystyle\int \frac{x}{1+x^2}\, dx$

Proprietà generali dell'integrale

$$\int \frac{d}{dx}[f(x)]\;dx \;=\; f(x)+C$$

$$\int (f(x)+g(x))\;dx \;=\; \int f(x)\,dx \;+\; \int g(x)\,dx$$

$$\int \alpha\cdot f(x)\,dx \;=\; \alpha\cdot\int f(x)\,dx\,, \quad \alpha\in\mathbb{R}$$

$$\int \alpha\cdot f(x)+\beta\cdot g(x)\,dx \;=\; \alpha\cdot\int f(x) \;+\; \beta\cdot\int g(x)\,dx\,, \quad \alpha,\ \beta\in\mathbb{R}$$

Integrale di una potenza

Base

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}, \; n \neq -1$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x|, \; (n = -1)$$

Versione generalizzata

$$\int f^n(x) \cdot f'(x) \, dx = \begin{pmatrix} u = f(x) \\ du = f'(x) \, dx \end{pmatrix} = \int u^n \, du = \frac{u^{n+1}}{n+1} = \frac{f^{n+1}(x)}{n+1}, \; n \neq -1$$

$$\int \frac{f'(x)}{f(x)} \, dx = \begin{pmatrix} u = f(x) \\ du = f'(x) \, dx \end{pmatrix} = \int \frac{1}{u} \, du = \ln|u| = \ln|f(x)|, \; (n = -1)$$

Esempi

Esempio 1

> Calcolare $I = \int \left(\frac{1}{x^2} + \frac{1}{x} - 1 + 2x + 3x^2 + x^\pi \right) \, dx$

Soluzione

$$
\begin{aligned}
I &= \int \frac{1}{x^2} \, dx + \int \frac{1}{x} \, dx - \int 1 \, dx + \int 2x \, dx + \int 3x^2 \, dx + \int x^\pi \, dx \\
&= \int x^{-2} \, dx + \int \frac{1}{x} \, dx - \int dx + 2 \int x \, dx + 3 \int x^2 \, dx + \int x^\pi \, dx \\
&= \frac{x^{-2+1}}{-2+1} + \log|x| - \frac{x^{0+1}}{0+1} + 2\frac{x^{1+1}}{1+1} + 3\frac{x^{2+1}}{2+1} + \frac{x^{\pi+1}}{\pi+1} \\
&= \frac{x^{-1}}{-1} + \log|x| - x + \cancel{2}\frac{x^2}{\cancel{2}} + \cancel{3}\frac{x^3}{\cancel{3}} + \frac{x^{\pi+1}}{\pi+1} \\
&= -\frac{1}{x} + \log|x| - x + x^2 + x^3 + \frac{1}{\pi+1}x^{\pi+1} + C
\end{aligned}
$$

Esempio 2

> Calcolare $\quad I = \int x(1+x^2)^{999}\,dx$

Soluzione

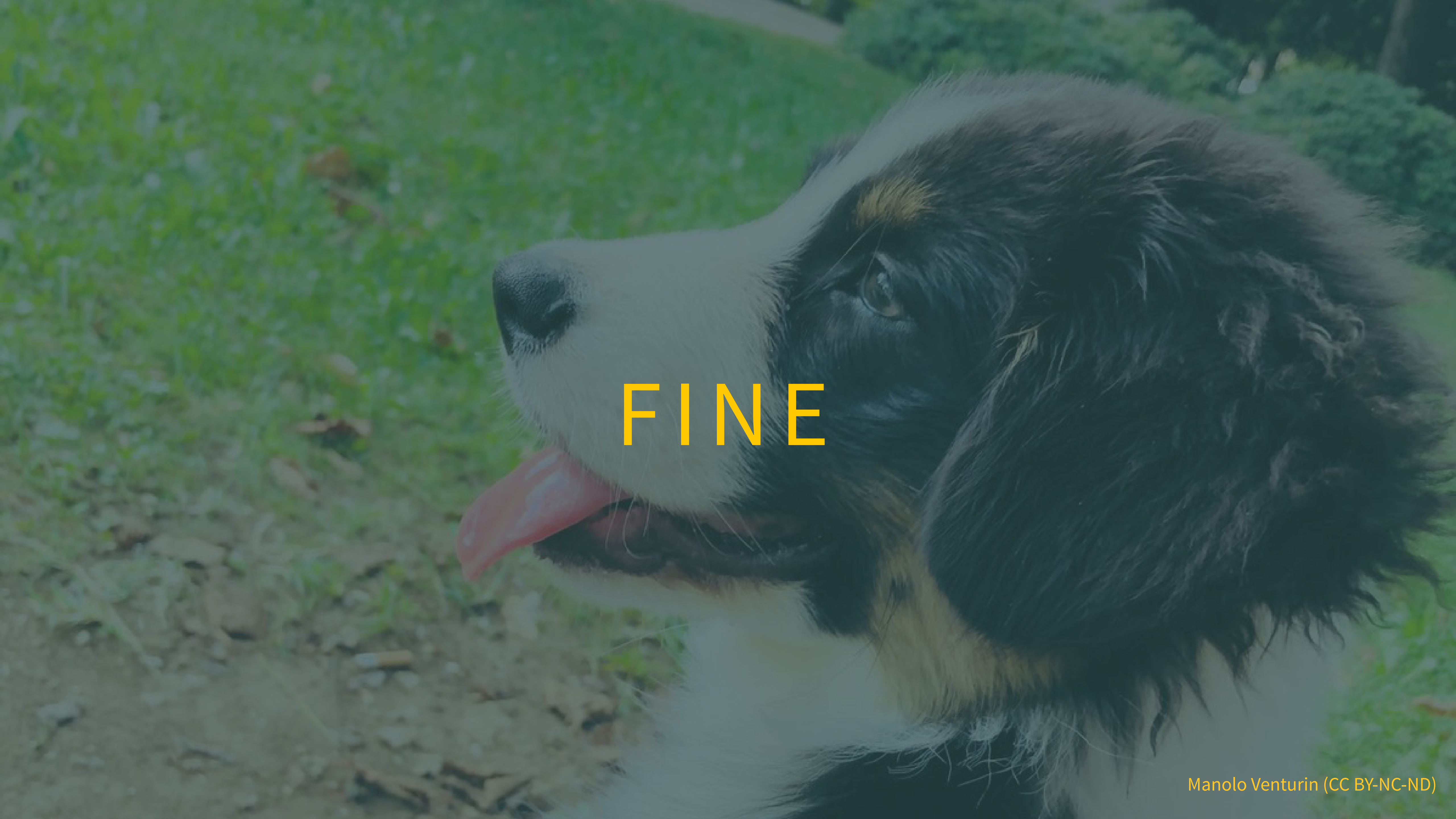
$$
\begin{aligned}
I &= \frac{1}{2}\int (1+x^2)^{999}\,2x\,dx \\
&= \begin{pmatrix} u = 1+x^2 \\ du = 2x\,dx \end{pmatrix} = \frac{1}{2}\int u^{999}\,du \\
&= \frac{1}{2}\,\frac{u^{999+1}}{999+1} = \frac{1}{2000}u^{1000} \\
&= \left(u = 1+x^2 \right) = \frac{1}{2000}(1+x^2)^{1000} + C
\end{aligned}
$$

Esempio 3

> Calcolare $I = \displaystyle\int \frac{x}{1+x^2}\,dx$

Soluzione

$$\begin{aligned}
I &= \frac{1}{2}\int \frac{1}{1+x^2}\,2x\,dx \\
&= \begin{pmatrix} u = 1+x^2 \\ du = 2x\,dx \end{pmatrix} = \frac{1}{2}\int \frac{1}{u}\,du \\
&= \frac{1}{2}\log|u| \\
&= \left(\, u = 1+x^2 \,\right) = \frac{1}{2}\log\left|1+x^2\right| + C
\end{aligned}$$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